Exam I

Problem ①

$\alpha = 0.4\times10^{-3}\ m^{-1}$

$\phi = 60^\circ$

$\omega = 10^9\ s^{-1}$

$\beta = 10^3\ m^{-1}$

$y(z,t) = e^{-\alpha z} A \cos(\omega t - \beta z + \phi)$

we use ⊖ sign because the wave is travelling in $+z$ direction.

$\rightarrow y(0,0) = A\cos(\phi) = A\cos(60^\circ) = A(\frac{1}{2}) = 8 \rightarrow A = 16$

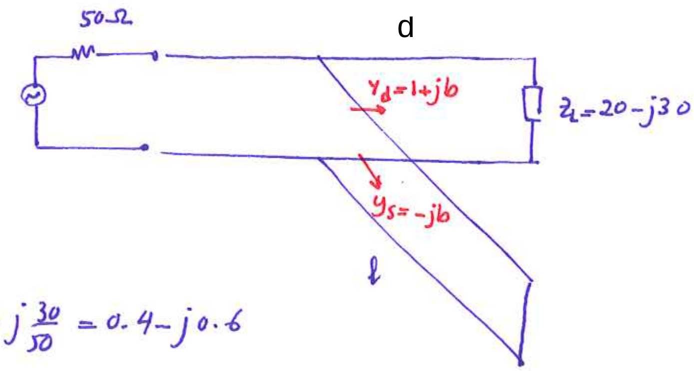
$\rightarrow y(z,t) = 16 e^{-0.4\times10^{-3} z} \cos(10^9 t - 10^3 z + 60^\circ)$

You may have started with $y(z,t) = e^{-\alpha z} A \sin(\omega t - \beta z + \phi)$. Then you would get:

$$y(z,t) = \frac{16}{\sqrt{3}} e^{-0.4\times10^{-3} z} \sin(10^9 t - 10^3 z + 60)$$

Both answers are correct.

Problem ②




$z_L = \frac{Z_L}{50} = \frac{20}{50} - j\frac{30}{50} = 0.4 - j0.6$

$d = (0.1725 - 0.1542)\lambda = 0.0183\lambda$ See the smith chart.

$l = (0.3515 - 0.25)\lambda = 0.1015\lambda$ For details study Example 2-12, p.88 in the book.

Problem ③ (a) $|D| = D_0 e^{-kR}$ (b) $\vec{\nabla}\cdot\vec{D} = \frac{1}{R^2}\frac{\partial}{\partial R}(R^2 D_0 e^{-kR}) = \frac{D_0}{R^2}(2Re^{-kR} - kR^2 e^{-kR}) = \frac{D_0}{R} e^{-kR}(2 - kR)$

(c) $\vec{\nabla}\times\vec{D} = 0$ (None of the terms is non-zero)

(d) $\vec{\nabla}(\vec{\nabla}\cdot\vec{D}) = \hat{R}\frac{\partial}{\partial R}\left(\frac{D_0}{R} e^{-kR}(2 - kR)\right) = \hat{R} D_0\left(2\frac{\partial}{\partial R}\left(\frac{e^{-kR}}{R}\right) - k\frac{\partial}{\partial R} e^{-kR}\right)$

$= \hat{R} D_0\left(2\left(\frac{-e^{-kR}}{R^2} - \frac{ke^{-kR}}{R}\right) + k^2 e^{-kR}\right) = \hat{R} D_0 e^{-kR}\left(-\frac{2}{R^2} - \frac{2k}{R} + k^2\right)$

(e) $\vec{\nabla}\times(\vec{\nabla}(\vec{\nabla}\cdot\vec{D})) = 0$ becaus $\vec{\nabla}\times\vec{\nabla}V = 0$ (curl of any gradient is zero)

(f) $\vec{\nabla}\cdot(\vec{\nabla}\times\vec{D}) = 0$ (divergence of any curl is zero)

# The Complete Smith Chart